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What's in a Name? A Critical Review of Definitions of Quantitative Literacy, Numeracy, and Quantitative Reasoning

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What's in a Name? A Critical Review of Definitions of Quantitative Literacy, Numeracy, and Quantitative Reasoning

Abstract

This article aims to bring together various threads in the eclectic literature that make up the scholarship around the theme of Quantitative Literacy. In investigating the meanings of terms like "quantitative literacy," "quantitative reasoning," and "numeracy," we seek common ground, common themes, common goals and aspirations of a community of practitioners. A decade ago, these terms were relatively new in the public sphere; today policy makers and accrediting agencies are routinely inserting them into general education conversations. Having good, representative, and perhaps even compact and easily digestible definitions of these terms might come in handy in public relations contexts as well as in other situations where practitioners need to measure and evaluate their own success or communicate their goals and practice to others. Finding such definitions is, as expected, a difficult task. We offer through our analysis a clarifying framework for practitioners looking to sharpen their definitions and for others who are not keen on finalizing definitions. More specifically, we argue that there is indeed a common thread among all the terms involved, that of a competence in interacting with myriad mathematical and statistical representations of the real world, in the contexts of daily life, work situations, and the civic engagement. Furthermore we propose that the knowledge content captured by the individual terms can be placed on a continuum (statistics-data-arithmetic-mathematics-logic).

Keywords

numeracy, quantitative literacy, quantitative reasoning, statistical literacy, statistical reasoning, mathematical literacy, mathematical reasoning, literacies; community of practice.

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Cover Page Footnote

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*What's in a name? that which we call a rose
By any other name would smell as sweet
Juliet, Act II, Scene II, Romeo and Juliet*

Introduction

Scholarship in the interdisciplinary world of quantitative literacy is thriving. The July 2015 editorial by Nathan Grawe celebrates 100,000 full-text downloads from the content of *Numeracy*, a journal dedicated to scholarship and education in quantitative literacy. July 2015 also marked the sixteenth issue of the journal; in its eight years, *Numeracy* has published approximately 150 papers. There are other outlets that scholarship around this theme will find friendly; PRIMUS for instance has published dozens of articles that revolve around quantitative literacy.

During all this time, scholars and educators have used terms such as *numeracy*, *quantitative literacy*, and *quantitative reasoning*, among others, sometimes interchangeably and sometimes paying attention to nuances and distinctions which were not all too clear to outsiders. In the North American context, *quantitative literacy* came to the fore with the Mathematical Association of America's calls for it; *numeracy* came under the spotlight with the publication of Paulos' nifty book *Innumeracy*; *quantitative reasoning* found its way to our lexicon when educators decided to steer clear from *mathematical thinking* as a terminal goal for a good education. In the rest of the world, the term *numeracy* is often understood as the locus of basic mathematical skills complementing a broader notion of literacy that all adults should be able to attain; the other phrases seem to appear much less frequently.

In this article we ask: How are these terms related? Are they synonymous? Or do they occupy neighboring areas of meaning? And what exactly do they mean? How can we define them? *Can* we define them?

These questions are of course not new. In particular, most who write about these themes in scholarly outlets offer some definitions before they proceed. Many who run programs or teach courses addressing these literacies attempt to delineate the territory before they launch. In a recent discussion among scholars, this was the central question (National Numeracy Network Annual Meeting, October 2014). A 2015 panel on quantitative literacy and democracy (MathFest, August 2015) began with a swift discussion of the same question. Two discussion lists of scholars recently saw lively conversations with over twenty participants on the very same thread. In this article, we mine through all of these, as well as a large chunk of the relevant literature, and aim to get to a common core.

Why are we interested in this question? Our community of practice thrived in the last decade, and enjoys today a well-respected journal, a handful of professional organizations, and regular, annual meetings. However, it is clearly

not an established academic discipline, at least not yet.¹ In order for it to develop further into a fully mature and self-sustaining community of practice, we should dedicate at least some energy on what our research objects and objectives ought to be:

Good science has to begin with good definitions. A research field can only be built and win legitimacy if it is differentiated from neighbouring fields. It can only impose its presence in the long term if it is able to establish its boundaries with other fields, even if those boundaries are, to some extent, fuzzy. This process necessarily means that the community of researchers must share in a given paradigm, in the sense given to the term by Kuhn (1970). A minimum level of consensus is needed on the definition of what the field is and is not, on the definition of the research object, and on its main themes, even if disagreements continue to exist on the fringes. [...] When there is no consensus on a paradigm, or at least on the main research object of the field, researchers tend to speak after one another, rather than to one another (Greenfield and Strickon 1986), and knowledge cannot be accumulated. Anarchy or epistemological ecumenism may lead to confusion, and the field does not progress. Its social legitimacy is therefore threatened. (Bryat and Julien 2000)

Thus it behooves us to explore systematically what we are doing. A natural starting point for such inquiry, as implied by the above quotation from researchers situating themselves within another young academic community, would be to seek clear definitions for the fundamental terms of our work. The philosophically or mathematically inclined would approve; Richard Feynman fans might disagree.²

Our thriving field of practice-based scholarship originated with policy documents, and our work continues to relate essentially to education policy. Today there is renewed interest in the topic among policy circles, including accreditation agencies, government bodies, and other public and private stakeholders. This makes the task at hand a lot more urgent. A question that could have been viewed only as an ideological issue, a concern arguably of purely academic concern, is today propelled forward by pressures from the policy world. We need a crisp and accurate definition for the terms we are using and we need them soon.

A desirable approach for a project seeking common ground, zeroing in on the common themes, goals, and aspirations of a community of practitioners, would be one that aims "to refine, expand, and characterize rather than to define, constrain,

¹ It is not clear that this is possible or even desirable. However, we will simply assume here that we do wish to ensure that our scholarly community continues to thrive, given that there is sustained intellectual and social need for it.

² "We can't define anything precisely. If we attempt to, we get into the paralysis of thought that comes to philosophers... one saying to the other: you don't know what you are talking about! The second one says: what do you mean by talking? What do you mean by you? What do you mean by know?" (Feynman et al. 1964, p. 8-2).

and circumscribe" (Rosenthal 1993). Still, a concrete description may be practically useful. Having good and perhaps even compact and easily digestible definitions of these terms might come in handy in public relations contexts as well as in other situations where practitioners need to communicate their goals and practice to others (journalists, policy makers, funding agencies) and even assess their own success (how do you measure something if you cannot even define it?)

Finding good definitions is, of course, a difficult task. In the following we offer a clarifying framework for practitioners looking to sharpen their definitions and for others who are not keen on finalizing definitions. We begin with a historical and data-driven overview of the main themes in question. We briskly survey some of the relevant literature, and we do not pretend that we are capturing the complete scenery; we aim simply to set the stage for what is to come. Then in the second section we devise a four-dimensional framework that represents different aspects of the cluster of ideas captured by these themes. We use this framework in following sections to analyze our themes. Along the way we propose a hierarchy for our three main terms and return to analyze various neighboring terms that several practitioners and segments of the society identify with our terms to determine more clearly the boundaries of the latter. After the main theme, we suggest interesting trends in the world of quantitative literacy that are tangential to our analysis: the analogies between literacy and numeracy, and QL as social practice. We include these brief discussions because we suspect such detours may help with definitions though we make no such connections here. We conclude with a pragmatic conclusion and close with a polemical endnote for those who are interested in asking some different, but related, questions.

What Can We Learn from History (and Google)?

Excellent overviews of the history of the terms in our title exist.³ We trace some of the historical steps here to gather data for our project of seeking good definitions.

The terms we focus on are almost all younger than a century. In Figure 1 we offer a frequency analysis, courtesy of Google books' Ngram Viewer, of the terms *numeracy*, *quantitative literacy*, and *quantitative reasoning*, as well as *mathematical thinking*, *mathematical reasoning*, *mathematical literacy*, *statistical thinking*, *statistical reasoning*, and *statistical literacy*. In our search we also

³ See for instance Madison and Steen (2008) or, for a quicker read, <http://www.carnegiefoundation.org/blog/a-brief-history-of-the-quantitative-literacy-movement/>; a careful timeline specific to the UK context can be found at <http://learning.gov.wales/resources/learningpacks/mep/numeracy/numeracy-and-society/numeracy-in-education-the-uk-context/?skip=1&lang=en>, both URLs were accessed last on August 28, 2015.

included *scientific thinking*, *scientific reasoning*, and *scientific literacy*, to provide a likely benchmark. We also include a plot, with the same terms excluding *numeracy*, in Figure 2.

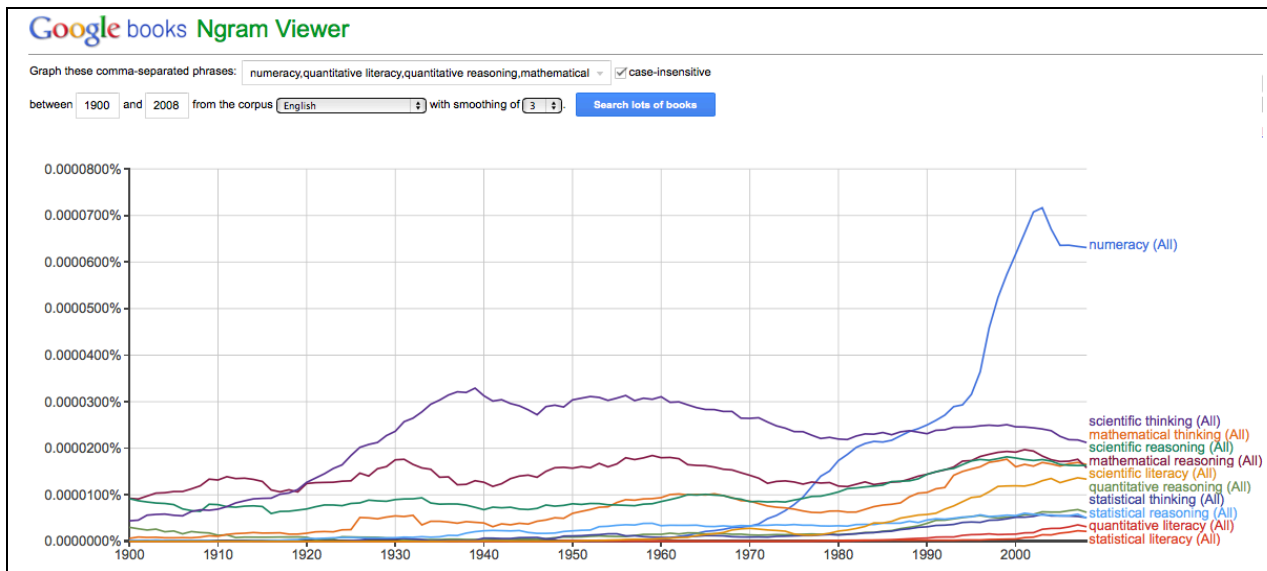


Figure 1. Prevalence of key terms in books through time. Courtesy of Google books Ngram Viewer, <http://books.google.com/ngrams>, August 28, 2015.

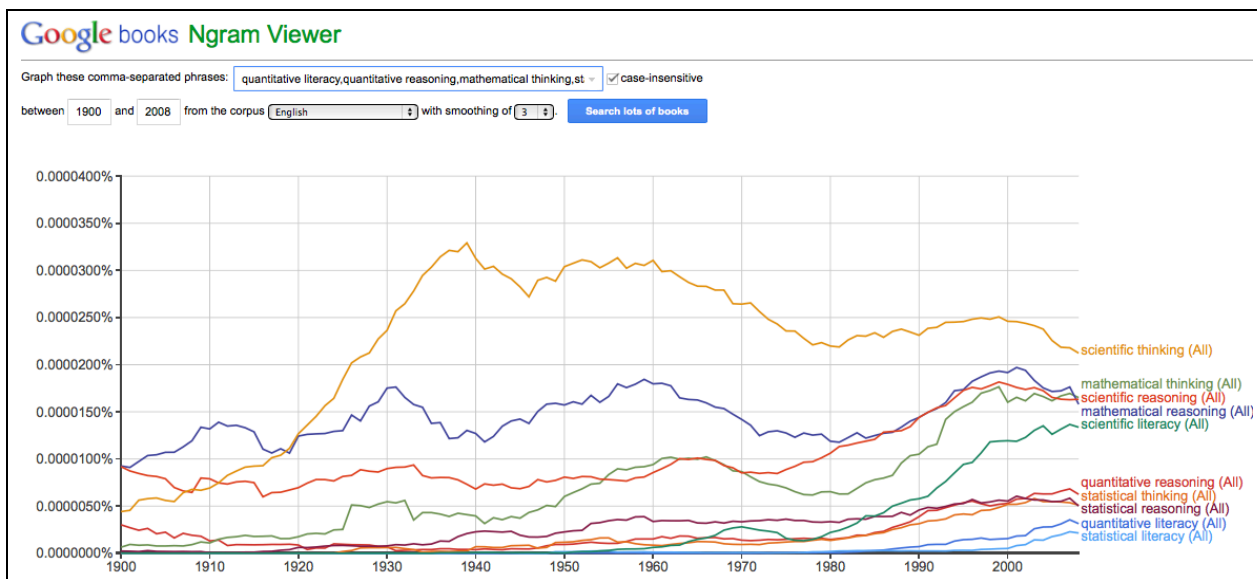


Figure 2. Prevalence in books of key terms, excluding *numeracy*, through time. Courtesy of Google books Ngram Viewer, <http://books.google.com/ngrams>, August 28, 2015.

It is clear from Figure 1 that among the terms we are going to focus on, the term *numeracy* is the one prevailing (in books included in the English corpus searched by Google books). It is also clear that this term originated rather recently.

Indeed the term *numeracy* seems to have originated in the Crowther Report (Crowther 1959). In this report we read:

Just as by "literacy", in this context, we mean much more than its dictionary sense of the ability to read and write, so by "numeracy" we mean more than mere ability to manipulate the rule of three. When we say that a scientist is "illiterate", we mean that he is not well enough read to be able to communicate effectively with those who have had a literary education. When we say that a historian or a linguist is "innumerate" we mean that he cannot even begin to understand what scientists and mathematicians are talking about. (p. 270)

The first thing we note here is that the term *numeracy* (used 24 times in the whole document) is offered in analogy with *literacy* (used 28 times). Thus it must be a different but parallel way of interacting with knowledge. We also see that there is a latent assumption that it is related to understanding; it is not really about procedural knowledge but rather comprehension. Then further down, we read:

It is perhaps possible to distinguish two different aspects of numeracy [...]. On the one hand is an understanding of the scientific approach to the study of phenomena - observation, hypothesis, experiment, verification. On the other hand, there is the need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind. (p. 270)

We see the parallels with scientific thinking; indeed it seems that numeracy is interpreted as the skill set required for one to be scientifically literate. Later in the text statistical fallacies are mentioned and compared to logical fallacies; surprisingly (at least for the mathematician and the scientist), the report authors situate the latter within the framework of *literacy* as opposed to *numeracy*.

Many scholars point next to the Cockcroft Report (Cockcroft 1982). This report is titled *Mathematics Counts*, and, as expected, *numeracy* (appearing 23 times) is much more prevalent than *literacy* (appearing 4 times). Again the parallel with literacy is made. Here, however, a shift in the meaning of the term (involving a departure from scientific thinking and the downgrading of the level of mathematical sophistication) is notable:

[W]e are in no doubt that the words, as commonly used, have changed their meaning considerably in the last twenty years. The association with science is no longer present and the level of mathematical understanding to which the words refer is much lower. This change is reflected in the various dictionary definitions of these words. Whereas the *Oxford English Dictionary* defines 'numerate' to mean 'acquainted with the basic principles of mathematics and science', *Collins Concise Dictionary* gives 'able to perform basic arithmetic operations'. (p. 11)

The report authors relate their reluctance to let go of expectations of sophistication and explicitly redefine the term:

We would wish the word 'numerate' to imply the possession of two attributes. The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life. The second is an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease. Taken together, these imply that a numerate person should be expected to be able to appreciate and understand some of the ways in which mathematics can be used as a means of communication. (p. 11, bold in the original)

This two-tiered description of *numeracy* has been adopted in many UK and Australian contexts.

Later, the report authors quote:

Statistical numeracy requires a feel for numbers, an appreciation of appropriate levels of accuracy, the making of sensible estimates, a commonsense approach to the use of data in supporting an argument, the awareness of the variety of interpretation of figures, and a judicious understanding of widely used concepts such as means and percentages. (p. 236)

The term *statistical numeracy* is not common today (see Table 1 for a frequency analysis of our keywords). Nonetheless as the authors of the Cockcroft Report explicitly endorse this definition, we will take it into consideration as we focus on defining characteristics.

Table 1.
Google search results for some of the terms used in the article

Search term	Results	Search term	Results	Search term	Results
<i>mathematical literacy</i>	411,000 / 1,420,000	<i>quantitative literacy</i>	139,000 / 1,310,000	<i>statistical literacy</i>	84,300 / 2,850,000
<i>mathematical reasoning</i>	483,000 / 2,470,000	<i>quantitative reasoning</i>	469,000 / 1,600,000	<i>statistical reasoning</i>	272,000 / 3,190,000
<i>mathematical</i>	115,000,000	<i>quantitative</i>	120,000,000	<i>statistical</i>	176,000,000

The search was done on August 28, 2015. The two numbers for the terms including multiple words indicate search results when terms were entered with or without quotes, respectively. In comparison, the term *numeracy* returned a much larger number (9,370,000 results) while the yields for the terms *mathemacy*, *matheracy*, and *statistical numeracy* were relatively insignificant (1250, 877, and 1560, respectively).

The roots of the North American conversation can be traced back to the 1950s when the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America (MAA) sponsored the production of curricular material for a “universal mathematics” course sequence, which, from a modern perspective, was targeting a quantitative literacy of sorts though the phrase had not yet materialized. Through the 1960s the CUPM did not

again focus seriously on these notions until its 1978 report, "Minimal Mathematical Competencies for College Graduates" (CUPM 1982). This report, though it would be relevant to a careful historical survey of the topic, was somewhat inconclusive and remained mostly inconsequential.

The 1980s saw more action. Notable were two NSF-funded, three-year, joint projects of the American Statistical Association (ASA) and the National Council of Teachers of Mathematics (NCTM), both titled Quantitative Literacy Project (QLP) (Scheaffer 1990; Burrill 1990). The first, funded in 1983, provided curriculum materials exploring elementary topics in data analysis, probability, simulation, and survey sampling; it served as the basis from which the strand in statistics was developed for the NCTM *Standards* (NCTM 1989). The second, funded in 1987, focused on in-depth professional development activities for secondary teachers. For an account of the role of the ASA in the QL movement, see Scheaffer (2003).

The term *numeracy* entered the American lexicon most categorically in 1988 with the publication of *Innumeracy: Mathematical Illiteracy and Its Consequences* (Paulos 1988). John Allen Paulos in this book identified *mathematical literacy* with *numeracy*; among these pages the phrase *numeracy* barely appears, and *mathematical literacy* seems even scarcer, but the terms *innumeracy* and *mathematical illiteracy* are used many times, and interchangeably. On page 3, Paulos defined innumeracy to be "an inability to deal comfortably with fundamental notions of number and chance" and, throughout the book, he gave examples that would overlap with most modern definitions of *quantitative literacy* and *statistical literacy*. Overall, his examples and his argument encompassed both, rather than siding with one or the other; for him, it seemed, the terms *numeracy* and *mathematical literacy* equally captured all.

Simultaneously with the publication of *Innumeracy* (Paulos 1988), the Consortium for Mathematics and Its Applications (COMAP 1988) published the first edition of *For All Practical Purposes: Mathematical Literacy in Today's World*, a textbook written in response to the question "What would you teach [undergraduate] students if they took only one term [or two terms] of math during their entire college career[s]?" We intend to analyze QL/QR textbooks in a different article.

Then in 1989, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (NRC 1989), a report authored by the Mathematical Sciences Education Board and the Board on Mathematical Sciences and Their Applications of the National Research Council, identified *numeracy* with *mathematical literacy*:

To function in today's society, mathematical literacy—what the British call "numeracy"—is as essential as verbal literacy ... Numeracy requires more than just familiarity with numbers. To cope confidently with the demands of today's society, one

must be able to grasp the implications of many mathematical concepts—for example, change, logic, and graphs—that permeate daily news and routine decisions—mathematical, scientific, and cultural—provide a common fabric of communication indispensable for modern civilized society. Mathematical literacy is especially crucial because mathematics is the language of science and technology. (p. 7)

Once again we note the significance of science here. Note also the emphasis on the need to go beyond familiarity with numbers. Just like literacy goes beyond the recognition of letters! We will come back to parallels with literacy soon.

The next major step was *Quantitative Reasoning for College Graduates*,⁴ the 1994 report (Sons 1996) by an MAA committee (Subcommittee on Quantitative Literacy Requirements) formed in 1989. Here we see the term *quantitative literacy* (used 166 times) explicitly used as a placeholder or a possible synonym for *numeracy* (used 4 times). The Preface of this report is a good overview of the previous engagement of American mathematics educators with the notion. Though the report was never officially adopted by the MAA, it was in some ways quite influential; several of its signatories were involved in the eventual founding of the Special Interest Group of the MAA on Quantitative Literacy in 2004.⁵

The first attempt at a description of *quantitative literacy* in this 1994 report comes after about forty uses of the term without any definition, in *Part I: Why Quantitative Literacy?* As the authors point out:

We have been speaking of mathematical attainments. The term "quantitative literacy" has so far appeared only in the title. Whether there is a real difference between "quantitative literacy" and "some significant proficiency in mathematical thinking and in the most useful elementary techniques that go with it" is a matter of debate. Sometimes the term "quantitative literacy" is a virtual euphemism for some level, usually ill defined, of accomplishment in mathematics. (How unfortunate that some people should consider it expedient to use a euphemism for "mathematics"!)

At other times "quantitative literacy" is used much more broadly, to include logic, linguistics, and other subjects that have at least a relatively formal character, even if they are seldom or ever taught in mathematics departments.

Immediately thereafter, the report authors declare that they will *not* provide a definition:

Here we shall adopt the point of view that "quantitative literacy" primarily concerns mathematics, broadly understood. It is not an entirely fortunate term. For one thing, much

⁴ This and several other noteworthy publications related to quantitative literacy are accessible from the MAA website: <http://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/quantitative-literacy>, accessed on November 15, 2015.

⁵ SIGMAA-QL is the Special Interest Group of the MAA on Quantitative Literacy. Founded in 2004, the group "aims to provide a structure within the mathematics community to identify the prerequisite mathematical skills for quantitative literacy (QL) and find innovative ways of developing and implementing QL curricula" (from <http://sigmaa.maa.org/ql/>, accessed on August 30, 2015).

of modern mathematics, even at elementary levels, is not distinctively quantitative; for another, "literacy" suggests both facility with letters and a possibly very low level of accomplishment. The term "numeracy" is shorter, at least. Most, if not all, of what will be said here will apply whichever reasonable interpretation of the term "quantitative literacy" is adopted.

Then they go on to make various arguments for the relevance and importance of adult quantitative literacy.

Incidentally this 1994 report uses the term *quantitative reasoning* in its title and then again approximately twenty-five more times. To the authors of the report, quantitative reasoning seems to be what quantitatively literate people are capable of doing; QR seems to be more of a process, while QL is a state.

The next significant effort that should be addressed is the 2001 publication of *Mathematics and Democracy: The Case for Quantitative Literacy* (Steen 2001). We share our analysis of this book in a later section. However let it suffice to say that this book made the case for QL as a fundamental component of civic education, thus bringing to the fore its connections to policy, politics, and democracy.

Much has happened since 2001, but these seminal reports and texts have been foundational to the community. With that we end our historical overview. But we are not done with Google!

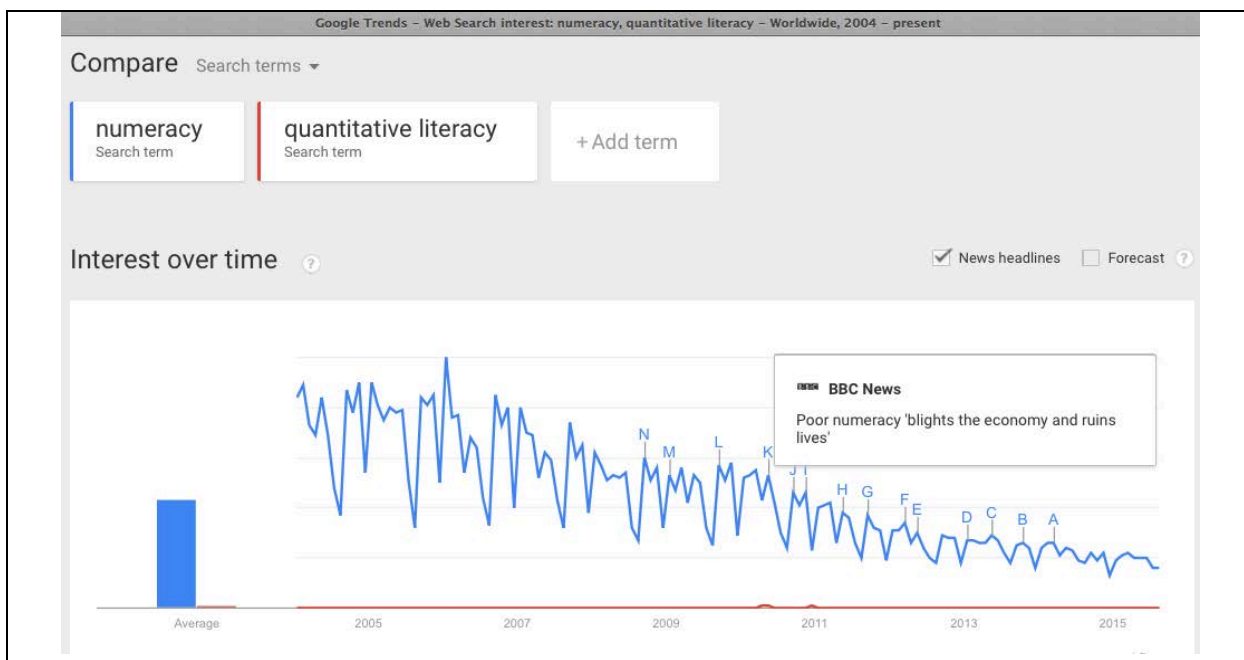


Figure 3. Google Trends search showing newspaper headlines using the terms *numeracy* and *quantitative literacy*. Data Source: Google Trends (www.google.com/trends), August 28, 2015. The same search displayed very clearly the almost mutually disjoint geographic loci of the users of these terms; while the term *numeracy* was predominantly used by UK- and Australian sources, the use of the term *quantitative literacy* seemed to be limited to the United States.

Nathan Grawe (2015) notes in his editorial that *Numeracy* articles using the term *numeracy* in their title have been downloaded significantly more often than those that use the term *quantitative literacy*. This is in line with what we saw in our Google Trends search (Fig. 3).

Perhaps, it would be a good idea for our community to think clearly about whether we want to dilute our efforts in promoting a single concept using different terms. Of course this assumes that the terms are all equivalent and that there is a unifying singular concept that they all capture. But is that so? We will offer a concrete answer to this question soon. More specifically, we will show that there is indeed a solid nucleus of coherent themes that are uniquely and individually conveyed by the terms *numeracy*, *quantitative literacy*, and their other friends. Each term captures a unique nuanced perspective, all the while sharing a concrete chunk of meanings with the rest.

A Theoretical Framework – Through a Pragmatic Methodology

In the preceding section we reviewed the *numeracy / quantitative literacy / quantitative reasoning* landscape in a mostly non-discriminating manner, though we have also pointed out a handful of differences in the terms and their uses. In the following sections, we will zero in on the individual terms and aim to circumscribe the notions they capture explicitly. In order to do that, in this section, we will seek some common threads and divergent perspectives that may help us distinguish between these terms.

We use for our main source of inspiration a recent discussion on a professional listserv dedicated to numeracy / quantitative literacy. On Wednesday July 1, 2015, Victor Piercey of Ferris State University, who is also the present chair-elect of SIGMAA-QL, asked for a “working definition of quantitative literacy.” Then followed a slew of responses, from many professionals who shared their own definitions, and the definitions their institutions have used. We have collected these responses, along with a handful of lists that were offered in the discussion. Figure 4 on the following page presents the word cloud formed by the collection of all the words that appeared in these definitions.

This is clearly not a systematic representation, but we can use it to note the concepts that bubble to the surface. These are, in no order of significance: *ability, apply, appreciation, arithmetic, basic, citizen, communication, concrete, confidence, context, critique, cultural, data, different, engage, evidence, future, habit, identify, information, literacy, logical, mathematics/mathematical, mind, needs, numeracy, problems, quantitative, reason, situations, sense, skills, statistics/statistical, use/using, work.*

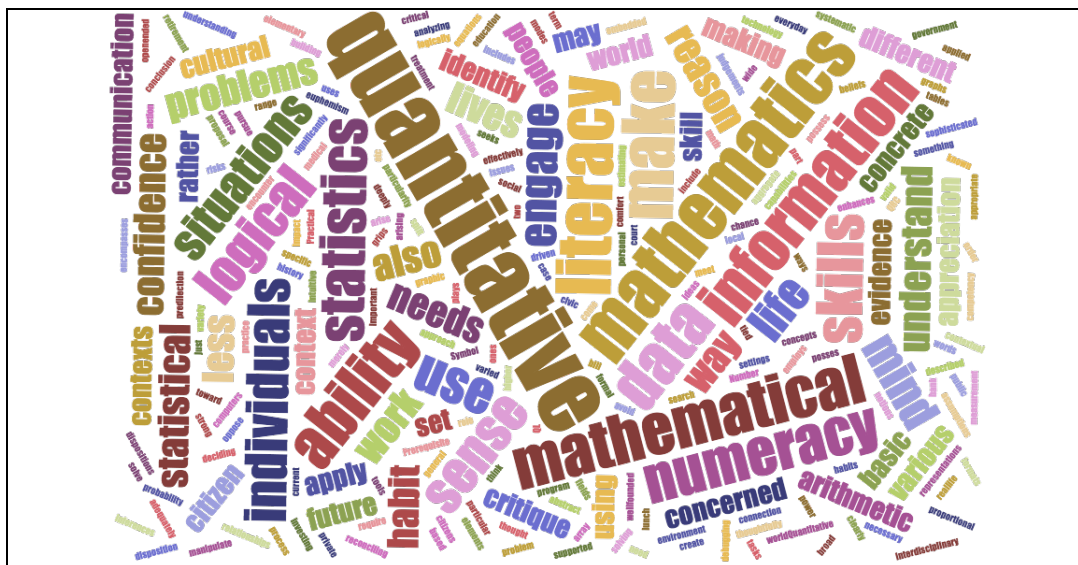


Figure 4. Word cloud formed by various “working definitions” of *quantitative literacy*. We should note that the definitions offered were mainly from U.S.-based individuals and institutions and included definitions for *quantitative literacy* and *quantitative reasoning*. Word cloud created by the word cloud generator <https://www.jasondavies.com/wordcloud> on August 10, 2015.

Organizing these words, we propose the following as significant dimensions in which we will analyze our key terms and their most common descriptions:

1. Quality of desired outcome: *a habit of mind* vs. *ability* vs. *skills*
2. Knowledge domain: *statistics* / *data* / *quantitative* / *arithmetic* / *mathematics* / *logic*
3. Display of expertise: *appreciate* / *communicate* / *critique* / *engage* / *reason* / *sense* / *understand* / *use* / *work*
4. Context: *citizen* / *context* / *evidence* / *future* / *information* / *problem* / *situations*

These may be reminiscent of Len Vacher’s expansive editorial (Vacher 2014) proposing a vocabulary matrix for the three keywords in our title (*numeracy*, *quantitative literacy*, *quantitative reasoning*). Vacher intended to map different loci of meaning used in the literature to these three terms (Table 2). In doing so he aimed to capture four dimensions: *cognitive content*, *cognitive ability*, *cognitive process*, and *cognitive attitude*. Our framework aims to work somewhat orthogonally to the one Vacher offers. We use our four-dimensional framework to extract the significant components of each of our key terms. We believe that our framework captures Vacher’s *cognitive content* (which aligns roughly with our *knowledge domain*) and his *cognitive process* (which aligns roughly with our *display of expertise*), while emphasizing that *context* and *type of expertise* are also significant components for the distinctions between the various terms.

Table 2.
A vocabulary matrix for “numeracy,” quantitative literacy,” and “quantitative reasoning” proposed in Vacher (2014).

	numeracy	quantitative literacy	quantitative reasoning
WS1. Skill with numbers and mathematics.	x		
WS2. Ability to read, write and understand material that includes quantitative information such as graphs, tables, mathematical relations, and descriptive statistics.	x	x	
WS3. Coherent and logical thinking involving quantitative information such as mathematical relations and descriptive statistics.		x	x
WS4. Disposition to engage rather than avoid quantitative information, using one’s mathematical skills and statistical knowledge in a reflective and logical way to make considered decisions.	x	x	x

WS = Word sense, as gleaned from study of *Word Net*, the online lexical database, <https://wordnet.princeton.edu/>

What is Numeracy?

*How do you solve a problem like Maria?
 How do you catch a cloud and pin it down?
 How do you find the word that means Maria?
 How do you keep a wave upon the sand?
 How do you hold a moonbeam in your hand?*

Maria (The Nuns), *The Sound Of Music*

As we have seen, the term *numeracy* originated in a policy document, the 1959 Crowther Report. However most refer to the follow-up report from 1982 (the Cockcroft Report) for a concrete definition for *numeracy*, more specifically its two-fold description for it:

1. an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life.
2. an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease.

The original 1959 report listed the scientific method as a component of numeracy, but, to this day, it remains mostly alone in that regard (though Paulos,

1988, does connect innumeracy with pseudoscience). Therefore we will not account for that possible theme in this work.

In Appendix A.1, we provide a selection of definitions that have been suggested and used through the years for the term *numeracy*. Analyzing these, we can see indeed that there is not much agreement as to what numeracy might mean. In particular while some authors and institutions take numeracy to involve reasoning and describe it as a habit of mind (see for instance AAC&U 2007), others tend to identify it with certain assessable skills (see for instance UNESCO 2006). We have to make some choices as we aim to order this complex system of ideas.

In the end we choose to conclude that in terms of our four dimensions, *numeracy* can be described as follows:

1. Quality of desired outcome: *an ability* to make use of some *skills* (basic⁶)
2. Knowledge domain: *mathematics/mathematical* and *arithmetic/quantitative* and *logical* (arithmetic-mathematics-logic spectrum)
3. Display of expertise: *understand* and *appreciate* and *cope* (passive, reactive)
4. Context: *information* and *practical situations* (daily life, possibly work)

This description, though not aligned perfectly with all possible definitions of numeracy out there, resonates with the common uses of the term. As Figures 1 and 3 show, the term *numeracy* appears significantly more often than the other alternatives, in all contexts except possibly the U.S. This may partially be because the nuances of the other terms are not as relevant to developing nations; *numeracy*, at its simplest seen as basic proficiency in mathematics that is useful in daily life, is a sufficiently challenging goal. For instance, the Global Partnership for Education, which lists *numeracy* as one of its top ten focus areas⁷ explicitly identifies it with math skills and labels the problem a “global math crisis.” The discourse focuses on “reading and math,” “literacy and numeracy.” The brevity and direct analogy with *literacy* must have a certain appeal.

What is Quantitative Literacy?

In contrast to the global dominance of the term *numeracy*, much of the U.S.-based scholarship seems to favor the terms starting with *quantitative*. *Quantitative*

⁶ “Basic” here and in the rest of the article does not imply “simple” or “unsophisticated.” In particular, a competence in numeracy might require quite sophisticated skills; see for instance the description from the National Numeracy Network website quoted in the section on QL / QR / QS / QT. In this framework, “basic” simply corresponds to the foundational level involving certain teachable and assessable skills.

⁷ See <http://www.globalpartnership.org/focus-areas>, last accessed on September 1, 2015.

literacy is endorsed as a worthy goal for education by various U.S.-based bodies, such as the Mathematical Association of America (MAA) and Association of American Colleges and Universities (AAC&U) (2007). Thus we next explore this term.

To do so, we could once again look at the discussion thread mentioned above, or go deeper into the difficulties and nuances (or “the many different faces,” Madison 2003) of quantitative literacy. For the sake of this section we focused instead on another classic in the field: *Why Numbers Count: Quantitative Literacy for Tomorrow’s America* (Steen 1997). Figure 5 shows the results of the textual analysis we ran.

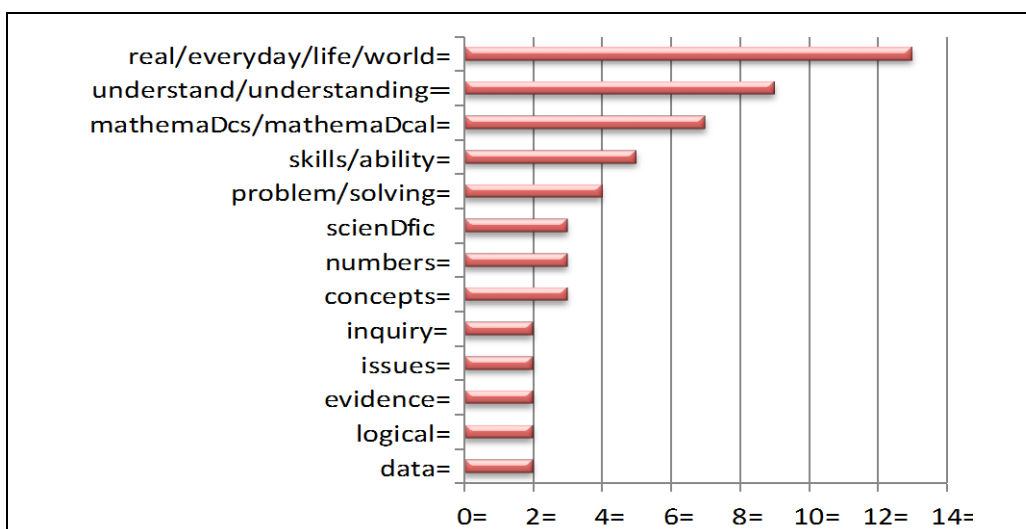


Figure 5. Word frequency analysis of definitions for QL from *Mathematics and Democracy* (Steen 1997). Source: <https://www.stolaf.edu/other/extend/Numeracy/defns.html>, accessed on September 1, 2015. The link features mostly comments on QL but also some reflections on numeracy and other Q-words. The latter were removed to avoid confounding effects. Analysis used <http://textalyser.net/>, on September 1, 2015.

The frequency analysis, though a somewhat dull tool, shows us the emphasis the contributors to *Why Numbers Count* put on a range of themes: *data, everyday, evidence, inquiry, life, logical, mathematics / mathematical, problem solving, real world, scientific, understanding*. Our analysis of the fifteen individual essays that make up *Mathematics and Democracy* (Steen 2001) aligns with this depiction. *Quantitative literacy* in these two edited volumes encompasses *confidence with numbers, appreciation for mathematics, ability to interpret data, to think logically, to make decisions logically, and to use mathematics in context*. This casts quite a wide net! However it is mostly representative of the meanings loaded on the term *quantitative literacy* by many contemporary users; see for instance

Mayes et al. (2013), which explicitly argues that QL encompasses numeracy, measurement, proportional reasoning, basic probability and statistics.

Including the listserv definitions for *quantitative literacy* into this mix, as well as several others included in Appendix A.2, we conclude that, in terms of our four-dimensional framework, *quantitative literacy* can be described as follows:

1. Quality of desired outcome: *ability and habit of mind* (intermediate to advanced)
2. Knowledge domain: *data, mathematics/mathematical, arithmetic/quantitative and logical* (data-arithmetic-mathematics-logic spectrum)
3. Display of expertise: *analyze, appreciate, decide, understand, use* (active, reactive)
4. Context: *citizen, information, practical situations* (daily life, work, civic life)

A Hierarchy of Terms?

So far we have focused on QL, but anybody delving into the literature we have been scrutinizing, both scholarly and institutional, will see many other Q-terms floating about. Besides *Quantitative Literacy* (QL), often used are *Quantitative Reasoning* (QR), *Quantitative Skills* (QS), and *Quantitative Thinking* (QT). In an earlier article (Karaali et al. 2010), we have used “QL / QR / QS / QT” to capture the whole gamut. In our brief analysis of the listserv thread mentioned earlier as well as in most of the literature, we see that often authors or institutions will pick one and stick with it. All of this makes one wonder if there is much to be gained from trying to distinguish these terms from one another. Do these terms signify practically the same concept? Can we then use one in place of another? Are they all interchangeable?

Vacher (2014) points to several articles and documents where these terms are explicitly identified with one another; we have also seen this treatment in policy documents as well as in textbooks, often with *numeracy* included in the mix. However it is reasonable to assume that there might be meaning differences whenever there are multiple terms in use. Indeed Vacher offers one perspective on possible differences through his vocabulary matrix, reproduced in Table 2. (Also see Appendix A.3 for distinct definitions for QR).

Other users of these terms are also aware of the nuances that differentiate them. Some, aligned with the 1994 MAA report (Sons 1996), see functional differences and choose to separate the state of QL from the process of QR. Others make more hierarchical distinctions. Travis Brown, the director of the Quantitative Skills Center at Pomona College, reminds us: “Being very good at

hammering a nail (QS) is not the same as building a house (QR)” (Brown 2014). A clear perspective is promoted on the National Numeracy Network website⁸:

Some call it **Numeracy**, an expression first used in the UK's 1959 "Crowther Report" to include secondary school students' ability to reason and solve sophisticated quantitative problems, their basic understanding of the scientific method, and their ability to communicate at a substantial level about quantitative issues in everyday life. Others call it **Quantitative Literacy (QL)**, and describe this comfort, competency, and "habit of mind" in working with numerical data as being as important in today's highly quantitative society as reading and writing were in previous generations. Still others refer to it as **Quantitative Reasoning (QR)**, emphasizing the higher-order reasoning and critical thinking skills needed to understand and to create sophisticated arguments supported by quantitative data.

In other words, this perspective argues, there is a hierarchy: At the foundational level is *numeracy*, perhaps identifiable with *quantitative skills*, with its emphasis on arithmetic skills and comfort with numbers. Then comes *quantitative literacy*, the fluency to comprehend quantitative information one confronts on a daily basis. Finally there is the summit, *quantitative reasoning*, which builds upon the former qualifications and captures the higher-order skills that are required to also be critical consumers of quantitative arguments.

This is indeed a neat perspective, and it is easy to accept. In fact, such a hierarchy could be easy to work with and advocate, as it can align well with educational levels. For instance *numeracy* could be what educators should focus on in the primary / elementary years. Middle / intermediate school education could initiate the emphasis on *quantitative literacy*. Finally in high school and possibly in higher education, we could cap things off with an explicit focus on *quantitative reasoning*. We will adopt this perspective in the rest of this paper.

For *quantitative literacy* to be an intermediate step in such a hierarchy, we need to make sure that it captures all that *numeracy* does. Many users of the term *quantitative literacy* do not specifically emphasize logic, but most definitions of *numeracy* and *quantitative reasoning* do. This could be a difficulty. However, going back to *Mathematics and Democracy* (Steen 2001), we see that the original conversations around *quantitative literacy* did include logic and reasoning. When we include “decision making” in *quantitative literacy*, we have to at least assume that those decisions will be made logically. We will come back to logic soon.

Another possible counterargument against this hierarchy is reflected in Vacher's vocabulary matrix, reproduced in Table 2. The main issue is that *numeracy* explicitly captures some mathematical skills and *quantitative literacy* and *quantitative reasoning* do not. Of course if we dig a bit deeper, we can see that the minimum mathematical skills captured by *numeracy* are simply assumed in the more sophisticated Q-notions. (Analogously, consider how one has to be

⁸ <http://serc.carleton.edu/nnn/resources/index.html> accessed on August 30, 2015

able to decipher the letters in an alphabet to arrive at reading comprehension.) Therefore without contradicting other authors, we can claim that *numeracy* could be a subset of (or a prerequisite for) the higher order Q-notions.

Our hierarchical perspective may remind readers of the work of Maguire and O'Donoghue (2004) who outline three stages of sophistication in their study of numeracy: formative, mathematical, and integrative. Interested readers may follow up on the above reference and other sources on adult literacy and numeracy that emphasize these three levels (see for instance AIR 2006).

What is Quantitative Reasoning?

In Table 1, we observe that between our two Q-terms, “quantitative reasoning” is indeed more popular. This may perhaps be partially explained by noting that the Educational Testing Service labels one of the three parts of its Graduate Record Examination Quantitative Reasoning. The ETS distinguishes between numeracy and problem solving.⁹ It is plausible that in their framework, *numeracy* together with *problem solving* makes up *quantitative reasoning*. Analogously, international adult literacy surveys (developed in collaboration with ETS) eventually dropped *quantitative literacy* as a dimension and replaced it with *numeracy* and *problem solving*, presumably capturing more (Thorn 2009).

In the previous section we argued for a hierarchical perspective for the three main terms we have been focusing on. In this hierarchy, QR was at the summit, a level of mastery and sophistication that went beyond the other two terms. We are now ready to offer a summary of where we see *quantitative reasoning* in terms of our four dimensions:

1. Quality of desired outcome: *habit of mind* (advanced)
2. Knowledge domain: *statistics/statistical, data, mathematics/mathematical, arithmetic/quantitative* and *logical* (statistics-data-arithmetic-mathematics-logic spectrum)
3. Display of expertise: *analyze, appreciate, critique, decide, understand, use* (active, reactive, proactive)
4. Context: *citizen, information, practical situations* (daily life, work, civic life)

Other Neighbors: *Mathematical Literacy, Matheracy, Mathemacy*

The term *mathematical literacy* dates further back than any of the other terms we are interested in (except possibly *statistical literacy*; more on that soon). A 1935

⁹ <http://www.ets.org/s/research/29836/>, last accessed on September 1, 2015

report to the Social Science Research Council (Brigham 1935) claims that the Scholastic Aptitude Test, first administered in 1926, offers “a measurement of *“mathematical literacy,”* an informal, intuitive, but highly speeded test involving the fundamentals of arithmetical, algebraic, and some elementary geometric thinking” [quotation marks and italics from the original]. Looking at this quotation and the contents of the test it refers to, it is clear that the origins of the term directly associate it with competence in elementary mathematics. In particular, though the arithmetic competence involved may be almost identified with *quantitative skills / numeracy*, we see that the mathematically literate are also required to be competent in some algebraic and geometric thinking.

We see this nuanced association of the term *mathematical literacy* with numeracy later in the twentieth century as well. In *Innumeracy* (Paulos 1988), for instance, the term *numeracy* is identified with *mathematical literacy*. But the focus is not solely on arithmetic. Some of his examples reflect some early algebraic thinking¹⁰ and logic.¹¹ A geometric dimension is explored more explicitly in some of the textbooks we explored, though it is mostly found in texts that aim for “mathematical appreciation” as well as “mathematical literacy.”

The case of geometry is in fact an intriguing one. A discipline that for centuries was at the heart of mathematics education, geometry today seems to have limited appeal for the proponents of *numeracy* and the accompanying Q-terms. When we seek out the historical justifications for geometry, we see that they originated from either practical concerns (how do I know where my field ends and where yours starts each year after the Nile floods?) or rational thinking arguments (geometry aka the axiomatic method is how rational thinkers reach universal truth). Some, like Johannes Kepler, went further and claimed divine justification.¹² For those not living by the whims of the Nile and possibly not impressed sufficiently by divine preference arguments, geometry today does not seem to hold much interest.

Of course the situation is not that simple. Euclid’s *Elements* until the twentieth century remained as the centerpiece of mathematics education, and classical geometry can still be a tool to help teach logical argument or the axiomatic methodology of deductive logic. Given the relative unpopularity of this path these days, we will also point out that a solid geometric training is the first

¹⁰ e.g., involving flexibility using percentages in the context of ordering food (Paulos, p. 97); understanding sales and comparing various deals involving price cuts (Paulos, p. 163).

¹¹ e.g., two versions of the prisoner’s dilemma (Paulos, p. 138-139)

¹² “Geometry is one and eternal, shining in the mind of God. That share in it accorded to men is one of the reasons that humanity is the image of God” (Kepler 1618, as cited in Gilder and Gilder 2004, p. 107).

step to *spatial reasoning*, a significant skill set that engineers, architects, and various others who incorporate visual design in their craft would benefit from. But what should the “typical person on the street” know about geometry?

Surely the person on the street should be able to park a car, guess whether a motor vehicle will or will not be able to squeeze in between two others, figure out whether the couch that needs to go down the stairs can actually go down the stairs without standing upright on its one side. Thus at the very least, transformation geometry, geometry that focuses on how shapes can be flipped, rotated and reflected around axes, seems like it could be a useful life skill. And it would naturally fit within *mathematical literacy*. Would it fit into any of the Q-terms?

Our claim is that it would not. The Q-terms overwhelmingly focus on the numerical/logical aspects of mathematics. The main visual component of the Q-terms is in graphical representations of data. Being able to use and possibly improve upon flow chart representations of processes and algorithms may also fit the bill. However geometry does not seem to come up.

Thus it is plausible that the term *mathematical literacy* is distinct from the locus of terms that are captured by *numeracy* and its Q-friends. Another corroborating evidence is that *mathematical literacy*, being specifically tied to a discipline, also aims for an aesthetic dimension, an appreciation of the power and beauty of that discipline, besides the mainly utilitarian approach the more discipline-independent terms might exclusively inspire. We will see this again in *statistical literacy*.

We should also mention here that various authors have approached *mathematical literacy* as a goal from different directions. There are some unique terms proposed in some of these contexts, such as *mathemacy* (e.g., Skovsmose 1998, 2005) and *matheracy* (e.g., D’Ambrosio 1999, Rosa and Orey 2015). It is important to note that these authors intend these terms to go beyond the discipline of mathematics and into the interactions with society. For instance Ole Skovsmose (1998, p. 195) defines *mathemacy* as “an integrated kind of competence including different forms of reflection (mathematics-oriented, model-oriented, context-oriented and lifeworld-oriented reflections).” Later he adds that it “must contain mathematical as well as reflective elements. [...] As an idealized notion, *mathemacy* must also include reflections on (mathematical) knowledge in action. [...] *Mathemacy* includes the hope of critical mathematics education that [...] address[es] the paradox of reason and [...] develop[s] a critical citizenship” (Skovsmose 2005, p. 188). Thus we see *mathemacy* unequivocally placed within the context of critical (mathematics) education. *Matheracy* on the other hand is defined in an ethnomathematical framework, as “the domain of skills, strategies, and competencies that empower students to be mindful of the way in which members of distinct cultural groups explain their beliefs, traditions, myths, symbols, and scientific and mathematical knowledge” (Rosa and Orey 2015, p.

593) though the authors also offer a simpler definition: “the capacity [...] to interpret and analyze signs and codes in order to propose models and to find solutions for problems faced daily.” (p. 587)

If we were to drastically simplify the above approaches to *mathematical literacy*, we’d say that mathemacy is more politically motivated while matheracy is more culturally oriented. In either case, however, we see that the concerns of those authors are not limited to numerical skills; geometry explicitly comes into play (e.g., wine producers using geometric schemes; Rosa and Orey 2015, p. 596). But more importantly these authors are deeply concerned with equity, justice, and the cultural repercussions of mathematics around the world. Of course, these concerns resonate significantly with recent work done in literacy, as “[c]ompeting definitions of literacy are really a form of cultural politics” (Apple 1992). Again, we will come back to literacy later.

Taking these closely related concepts in connection with the OECD / PISA definition of *mathematical literacy*,¹³ we can see clearly that many of the themes we tackled earlier are captured by most conceptions of *mathematical literacy* and its friends. In particular, if we follow the many authors encouraging a move from a functional definition of *mathematical literacy* to a critical one,¹⁴ we can see the resonances much more clearly.

Other Neighbors: **Statistical Literacy, Statistical Numeracy, Statistical Reasoning**

As mentioned earlier, the term *statistical numeracy* was defined in the Cockcroft report as “*a feel for numbers, an appreciation of appropriate levels of accuracy, the making of sensible estimates, a commonsense approach to the use of data in supporting an argument, the awareness of the variety of interpretation of figures, and a judicious understanding of widely used concepts such as means and percentages*” (Cockcroft 1982, p. 236). The lack of frequency of the particular term notwithstanding (recall Table 1), the idea of *statistical numeracy* does capture a cloud of themes distinct from the ones discussed earlier.

The term also has a comparatively extended history. Already in 1943, we find Douglas Scates (1943, p. 69) defining *statistical literacy* as “*the ability to read*

¹³ “*an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen*” (OECD 2003, p. 24)

¹⁴ “*Functional mathematical literacy pertains to the applications of mathematics required in practical contexts, while critical mathematical literacy has the capacity to change the ways of thinking and even challenge society.*” (Wood et al. 2012, pp. 4-5)

in 1883.¹⁶ Originally meaning “the ability to read and write,” today the word has many other connotations. Merriam-Webster lists the following as synonyms: “erudition, knowledge, learnedness, learning, education, scholarship.” Nonetheless, the technical term today represents a particular cloud of ideas, the nuances of which are well captured by the following paragraph from UNESCO:

At first glance, ‘literacy’ would seem to be a term that everyone understands. But at the same time, literacy as a concept has proved to be both complex and dynamic, continuing to be interpreted and defined in a multiplicity of ways. People’s notions of what it means to be literate or illiterate are influenced by academic research, institutional agendas, national context, cultural values and personal experiences. In the academic community, theories of literacy have evolved from those focused solely on changes in individuals to more complex views encompassing the broader social contexts (the ‘literate environment’ and the ‘literate society’) that encourage and enable literacy activities and practices to occur. As a result of these and other developments, understandings in the international policy community have expanded too: from viewing literacy as a simple process of acquiring basic cognitive skills, to using these skills in ways that contribute to socio-economic development, to developing the capacity for social awareness and critical reflection as a basis for personal and social change. (UNESCO 2006, p. 147)

We see just the same complexity and the same evolution in *numeracy*! Indeed the report from UNESCO does mention the path our terms have taken, from a skills-based understanding to a much broader understanding of an individual’s place in the society.

Here we might take notice of the *multiple literacies* construct (see for instance Lankshear and Knobel 2003; Jewett 2011 is a shorter introduction) and consider the various different literacies it takes for an adult to navigate our modern world effectively. In Table 3 we explore a range of literacies that are proposed as significant components of a complete education in various contexts.

Given the results shown in Table 3, we can see that once we start thinking in terms of multiple literacies, *quantitative literacy* is no longer part of a two-sided analogy, but one among many competencies that today’s individuals and societies aspire to. Once we move to this perspective, the term *literacy* may open up to capture the whole spectrum of literacies. Indeed, in the end, the ultimate goal, if we need a singular cause, seems to be reading comprehension, writ large. This captures all contexts delineated by the individual literacies, and if we allow for text to encompass communication in all other media, the idea of literacy indeed becomes the common target. In this new context, *numeracy* may be viewed as “a component of literacy itself” (UNESCO 2006, p. 149).

¹⁶ <http://www.merriam-webster.com/dictionary/literacy>, September 12, 2015. Note that JSTOR locates earlier instances, for instance in Richard Grant White, “The Public-School Failure,” *The North American Review*: Vol. 131: No. 289 (December 1880), pages 537–550.

Table 3.
Google search results for different literacies.

Search term	Results	Search term	Results
<i>technology literacy</i>	305,000 / 16,800,000	<i>digital literacy</i>	1,520,000 / 14,600,000
<i>information literacy</i>	2,500,000 / 18,700,000	<i>visual literacy</i>	468,000 / 5,770,000
<i>financial literacy</i>	4,740,000 / 13,000,000	<i>environmental literacy</i>	236,000 / 4,500,000
<i>quantitative literacy</i>	139,000 / 1,310,000	<i>statistical literacy</i>	84,300 / 2,850,000
<i>mathematical literacy</i>	411,000 / 1,420,000	<i>scientific literacy</i>	440,000 / 3,110,000
<i>moral literacy</i>	19,400 / 32,600,000	<i>legal literacy</i>	151,000 / 9,170,000

The search was done on August 28, 2015. The two numbers for the terms including multiple words stand for search results when terms were entered with or without quotes, respectively. Recall that the term *numeracy* returned “about 9,370,000 results”. In comparison, a search for the term *literacy* on this same date yielded 33,100,000 results.

Quantitative Literacy as Social Practice

The relationship of *numeracy* to *literacy* is complex and intriguing, and the deeper we dig, the more we find. We thus have to admit that the nuances of this relationship go beyond the scope of our current project. We nonetheless note one idea from the extensive literature on *literacy* and invite the readers to engage with it in our context.

One of the most productive ideas in literacy studies of the last couple decades is the conception that *literacy* is always situated in a social and political context (Gee 1990; Luke 1991; Street 1995). This brings us to the growing body of work on “*numeracy as social practice*” (Nunes et al. 1993; Street 2005; Street and Baker 2006; Kelly et al. 2007; see also Frith and Lloyd 2016, in this issue). As Hamilton, Hillier and Tett argue in their Introduction to Hamilton et al. (2006), this approach is situated in contrast with a “*functional skills-based*” approach that focuses on “*deficit or lack*” and “*leads to narrow, reductionist definitions [...] and ignores aspects of learning that cannot be dealt with at the psychological or cognitive level*” (page 3). Stephen Reder, in the Foreword, writes:

The social practices perspective has grown out of, and in reaction to, an older and narrower psychological approach that focuses only on individual cognitive skills. The social practices approach helps us think about what it is that people know and do in everyday life, and helps to locate meanings, values and purposes within a basic skills framework. [...] literacy, numeracy and language are more than a set of skills or techniques; they are cultural practices shaped by the social and historical contexts in which they occur and by the meanings they have and the purposes they serve do their participants. This approach shifts the focus from narrowly functional and externally imposed definitions of literacy, numeracy, communication and language to more open and numerous definitions that focus on what people do with their basic skills, with whom, when and how. (page vii)

To some, this may seem like an inspiring but ultimately impractical approach. How is contextualizing *numeracy* in social practice going to help us? The book (Hamilton et al. 2006) takes this challenge head on. Besides delineating various practice and policy implications,¹⁷ the authors prove that the approach offers opportunities for introspection and reflection. Take for instance this point, again from the Foreword:

When governments or educational agencies report ‘problems’ related to large numbers of individuals having ‘substandard’ basic skills, we should ask for whom this is a problem, or more succinctly, whose problem it is. Is it a problem for the assessed person? Or is it a problem for the educational agency? For employers? Society? [A] ‘literacy problem’ is often best defined as a difference in expectations regarding the performance of literacy or numeracy practices. [W]hen learners score poorly in relation to a national [assessment], yet fail to report difficulties performing everyday tasks drawing on those same skills, the real problem may well be the discrepant perceptions about literacy or numeracy. (p. ix)

We underline here that “whose problem is it?” is a good question for our community of practice to keep in mind. Papen (2005) presents careful and insightful critiques of various policy documents via this approach.

A Pragmatic Conclusion

I shall not today attempt further to define the kinds of material I understand to be embraced within that shorthand description; and perhaps I could never succeed in intelligibly doing so. But I know it when I see it.

Justice Potter Stewart, concurring opinion in *Jacobellis v. Ohio* 378 U.S. 184 (1964).

In their attempt to describe their attitude toward the notion of mathematical explanation, philosophers Paolo Mancosu and Johannes Hafner quote William James in *The Varieties of Religious Experience*:

“Most books on the philosophy of religion try to begin with a precise definition of what its essence consists of. Some of these would-be definitions may possibly come before us in later portions of this course, and I shall not be pedantic enough to enumerate any of them to you now. Meanwhile the very fact that they are so many and so different from one another is enough to prove that the word ‘religion’ cannot stand for any single principle or essence, but is rather a collective name. The theorizing mind tends always to the over-simplification of its materials. This is the root of all that absolutism and one-sided dogmatism by which both philosophy and religion have been infested. Let us not fall immediately into a one-sided view of our subject, but let us rather admit freely at the outset that we may very likely find no one essence, but many characters which may

¹⁷ For an overview, see the first chapter, “Introduction: Social Practice of Adult Literacy, Numeracy and Language,” pages 1–18.

alternately be equally important in religion. If we should inquire for the essence of 'government,' for example, one man might tell us it was authority, another submission, another police, another an army, another an assembly, another a system of laws; yet all the while it would be true that no concrete government can exist without all these things, one of which is more important at one moment and others at another. The man who knows governments most completely is he who troubles himself least about a definition which shall give their essence. Enjoying an intimate acquaintance with all their particularities in turn, he would naturally regard an abstract conception in which these were unified as a thing more misleading than enlightening. And why may not religion be a conception equally complex?"

and suggest that readers replace "religion" with "explanation" (Hafner and Mancosu 2005). In this article, we avoided the temptation to do the same. This led us to what James would probably view as pedantry: we indeed enumerated several definitions (see Appendix A). However we did this for a purpose. We wanted to show that the notions of *numeracy*, *quantitative literacy*, and *quantitative reasoning*, all their complexities and nuances aside, "stand for [a] single principle or essence", that they simultaneously represent a collective of ideas and practices.

So have we done what we promised? Have we been able to capture the essential concepts of this decades-long conversation on *numeracy*, *quantitative literacy*, and their neighbors?

We believe we have succeeded, at least partially. More specifically,

1. We have determined that there is indeed a common thread, that of a competence in interacting with myriad mathematical and statistical representations of the real world, in the contexts of daily life, work situations, and the civic life.
2. We have determined the knowledge content captured by the individual terms and placed them on a continuum (statistics-data-arithmetic-mathematics-logic).
3. We have argued that the differences, if any are to be perceived, may be ascribed to
 - a) levels of sophistication (basic / intermediate / advanced),
 - b) action (understand / use / critique), and
 - c) engagement of mental faculty (algorithmic-procedure / critical-reason).

Furthermore,

4. We have attempted to delineate the boundaries between our terms with similar sounding terms. In particular we observed that discipline-oriented terms like *mathematical* and *statistical literacy* often incorporate some aesthetic expectations, somewhat in contrast to the ultimately utilitarian expectations of our terms.

However, we chose not to give final definitions for any of the terms we analyzed. It is clear that the wealth of definitions we have surveyed have been fruitful and constructive in different contexts, and so we have decided not to judge them against one another. We furthermore decided that instead of offering our own definitions, which would undoubtedly have to be idiosyncratic despite any sincere attempts at universality, we would share with readers a multitude of definitions in our appendix, along with our analysis, so that interested practitioners may pick and choose as they wish.

We hope that our work nonetheless helps clarify these notions and organizes the cluster of ideas involved. Our hope is that when we need a definition, we can pick one that addresses precisely what we want. Then we can move forward with our main task: to educate and to prepare future generations for the diversity of quantitative communication contexts they will find themselves in. Even bearing in mind that as our community evolves, our definitions will evolve as well, we hope that we have at least convinced our readers that there is a solid nucleus of coherent themes that are individually conveyed by the terms *numeracy*, *quantitative literacy*, and their other friends.

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Appendix A. Some definitions of numeracy, quantitative literacy, and quantitative reasoning

A.1. Numeracy

[Cockcroft Report 1982] ... equate *numeracy* with an ability to cope confidently with the mathematical demands of adult life. ... an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life... an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease.

[Paulos 1988] defines innumeracy as “an inability to deal comfortably with the fundamental notions of number and chance” (page 3). So Paulos implicitly defines numeracy as “an ability to deal comfortably with the fundamental notions of number and chance”.

[Steen 1997]¹⁸ *Numeracy* [...] is an aggregation of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively and autonomously in quantitative situations arising in life and work. (Ido Gal, cognitive scientist)

¹⁸ Definitions referencing [Steen 1997] in this appendix were taken from <https://www.stolaf.edu/other/extend/Numeracy/defns.html>, which in turn was adapted with permission from "Why Numbers Count: Quantitative Literacy for Tomorrow's America," [Steen 1997].

[UNESCO 2006] More recently, '*numeracy*' has been used to refer to the ability to process, interpret and communicate numerical, quantitative, spatial, statistical and even mathematical information in ways that are appropriate for a variety of contexts (Box 6.3). The term increasingly refers to a competence allowing more effective participation in relevant social activities (Evans 2000). [*N*]umeracy should be seen as a semi-autonomous area at the intersection between literacy and mathematics and address not only purely cognitive issues, but also students' dispositions and cognitive styles.

[AAC&U 2007] the habit of mind, competency, and comfort in working with numerical data. (*numeracy* = *QL* = *QR*)

[Follette and McCarthy 2013] the ability to reason using numbers, graphs, statistics, etc. in order to be an effective participant in modern society.

[Grawe 2014] the ability to apply quantitative evidence to arguments in broad contexts of personal and public life. (*numeracy* = *QL*)

A.2. Quantitative literacy

[Steen 1997] *Quantitative literacy* involves understanding the role of numbers in the world. It provides the ability to see below the surface and to demand enough information to get at the real issues. (Ted Porter, historian)

Beyond arithmetic and geometry, *quantitative literacy* also requires logic, data analysis, and probability.... It enables individuals to analyze evidence, to read graphs, to understand logical arguments, to detect logical fallacies, to understand evidence, and to evaluate risks. *Quantitative literacy* means knowing how to reason and how to think. (Gina Kolata, journalist)

Quantitative literacy can be defined as the level of mathematical knowledge and skills required of all citizens. It includes the ability to apply aspects of mathematics (including measurement, data representation, number sense, variables geometric shapes, spatial visualization, and chance) to understand, predict, and control routine events in people's lives. (John Dossey, mathematics educator)

Quantitative literacy requires one to understand the nature of mathematics and its role in scientific inquiry and technological progress; to grasp sufficient mathematics to understand important scientific and engineering concepts; and to possess quantitative skills sufficient for responding critically to scientific issues in the media and public life. (F. James Rutherford, physics educator)

The heart of *quantitative literacy* is real world problem solving--the use of mathematics in everyday life, on the job, and as an intelligent citizen. Problem solving must be both mathematically defensible and useful in the real world. (Henry Pollak, applied mathematician)

Quantitative literacy involves understanding the mathematical concepts and skills that are necessary for everyday life. It includes computation, interpretation, inquiry, and

application of mathematical concepts that are critical for life in the contemporary world. (Glenda Price, college provost)

Quantitative literacy involves reasoning with numbers (Jim Lewis); reading, interpreting and making simple applications (Carole Lacampagne); understanding operations on rational numbers (Jack Price); constructing and recognizing a sound argument (Keith Devlin); and understanding variability and how to quantify it (Gail Burrill).

[NCES 2002] the knowledge and skills required to apply arithmetic operations, either alone or sequentially, using numbers embedded in printed materials; for example, balancing a checkbook, figuring out a tip, completing an order form, or determining the amount of interest from a loan advertisement.

[Frith and Prince 2006] *Quantitative literacy* is the ability to manage situations or solve problems in practice, and involves responding to quantitative (mathematical and statistical) information that may be presented verbally, graphically, in tabular or symbolic form; it requires the activation of a range of enabling knowledge, behaviours and processes and it can be observed when it is expressed in the form of a communication, in written, oral or visual mode.

[AAC&U 2007] the habit of mind, competency, and comfort in working with numerical data. (*numeracy* = *QL* = *QR*)

[Grawe 2014] the ability to apply quantitative evidence to arguments in broad contexts of personal and public life. (*numeracy* = *QL*)

Esther Wilder, Lehman College/ CUNY¹⁹: the application of mathematical thought and knowledge to authentic, everyday issues (*QL* = *QR*)

Michigan State University: *Quantitative literacy* is the ability to formulate, evaluate, and communicate conclusions and inferences from quantitative information.

Michael Schuckers, St. Lawrence University: an improvable habit of mind for selecting and obtaining; analyzing and evaluating; implementing and communicating appropriate quantitative methods and evidence across a variety of contexts. These habits include methods for composition, analysis and critique of numerical arguments.

A.3. Quantitative reasoning

[Steen 1997] *Quantitative reasoning* [is] **an interpretive activity** that takes place within a deductively structured framework. It involves a tapestry of meaning provided by a warp of abstract patterns and a weft of context and story line. In quantitative reasoning, context provides meaning. (George Cobb, statistician)

¹⁹ Names attached to institutions signify definitions provided in the discussion thread mentioned earlier in the paper.

[AAC&U 2007]: the habit of mind, competency, and comfort in working with numerical data. (*numeracy* = *QL* = *QR*)

Nathan Grawe, Carleton College: the habit of mind to consider the power and limitations of quantitative evidence in the evaluation, construction, and communication of arguments in public, professional, and personal life.

Esther Wilder, Lehman College/ CUNY: the application of mathematical thought and knowledge to authentic, everyday issues (*QL* = *QR*)

Lisa Cooley, Piedmont Community College: The skill of interpreting logical, graphical or numerical information to develop solutions to real-life problems.

University of Virginia: *Quantitative reasoning* is correctly using numbers and symbols, studying measurement, properties, and the relationships of quantities, or formally reasoning within abstract systems of thought to make decisions, judgments, and predictions.

James Friedrich, Willamette University: the process of solving problems, drawing valid inferences, and understanding, formulating and disseminating appropriate arguments based on information subjected to quantitative analysis.